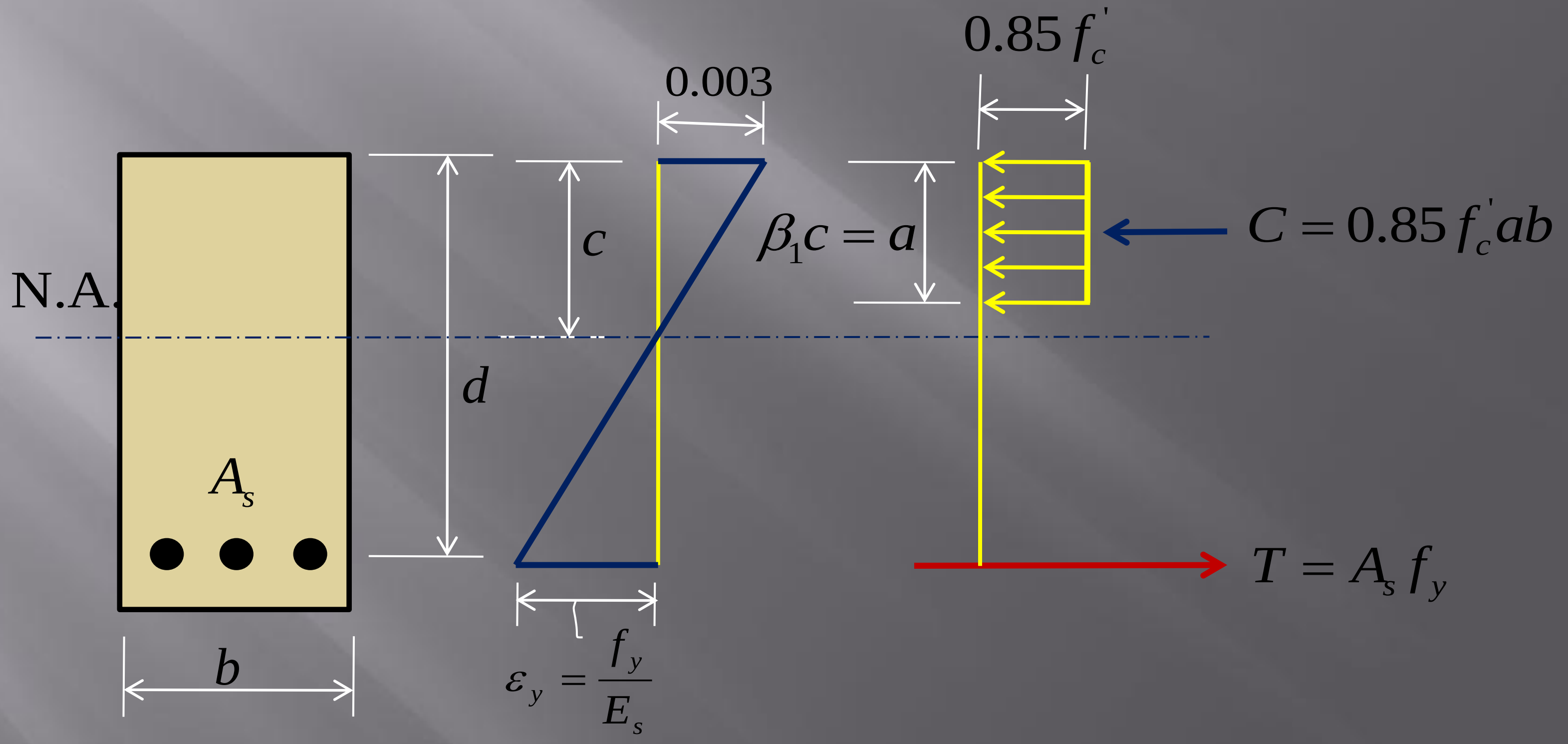
REINFORCED CONCRETE-I

(Strength Analysis of Beams..Contd)

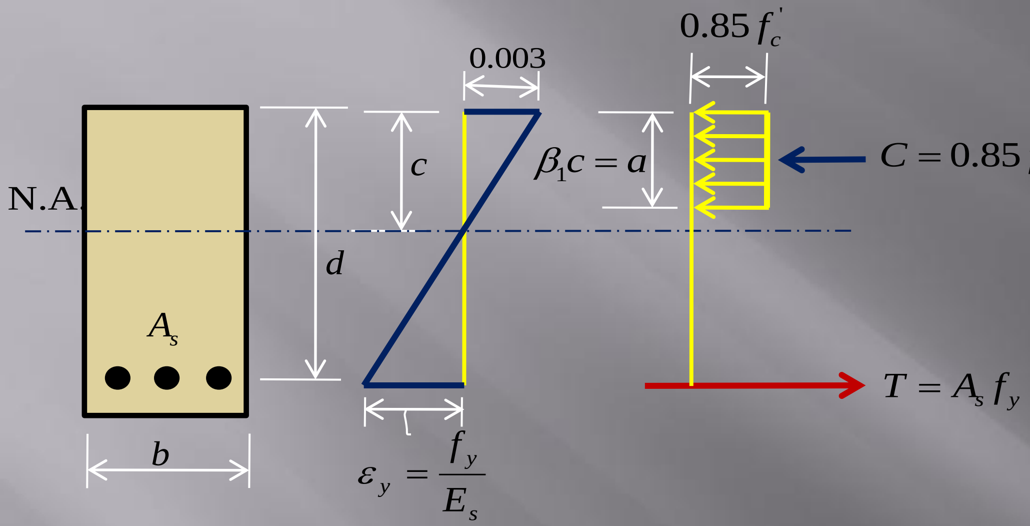
(According to SBC/ACI Code)

Balanced Design

In Balanced design, at ultimate load, the concrete and steel simultaneously reach to their ultimate (0.003) and yield strains ($\varepsilon_y = f_y/E_s$) respectively.



Balanced Steel Ratio



$$\because C = T \Rightarrow 0.85 f_c' ab = A_s f_y$$

$$\Rightarrow a = \frac{A_s f_y}{0.85 f_c' b} = \frac{\rho f_y d}{0.85 f_c'} \textbf{ where } \rho = \frac{A}{bd}$$

$$\because a = c\beta_1 \Rightarrow c = \frac{a}{\beta_1} = \frac{\rho f_y d}{0.85 \beta_1 f_c'}$$

$$\frac{c}{d} = \frac{0.003}{(0.003 + \varepsilon_y)}$$

$$\Rightarrow c = \frac{0.003d}{\left(0.003 + \frac{f_y}{E_s}\right)}$$

$$\therefore \frac{\rho f_y d}{0.85 \beta_1 f_c'} = \frac{0.003d}{0.003 + \frac{f_y}{E_s}}$$

$$\Rightarrow \frac{\rho_b f_y}{0.85 \beta_1 f_c'} = \frac{0.003}{0.003 + \frac{f_y}{200000}} \qquad (\because E_s = 2 \times 10^5 \text{ MPa})$$

$$\Rightarrow \frac{\rho_b f_y}{0.85 \beta_1 f_c'} = \left(\frac{600}{600 + f_y}\right)$$

$$\Rightarrow \rho_b = \left(\frac{0.85 \beta_1 f_c'}{f_y}\right)\left(\frac{600}{600 + f_y}\right)$$

Section Types Based on Ductility

$\rho = \rho_b \Rightarrow$ **Balanced section i..e. steel reaches yield when concrete crushes.**

$\Rightarrow$ **Called Balanced sections/members**

$\rho < \rho_b \Rightarrow$ **Ductile failure i.e. steel yields before concrete crushes**

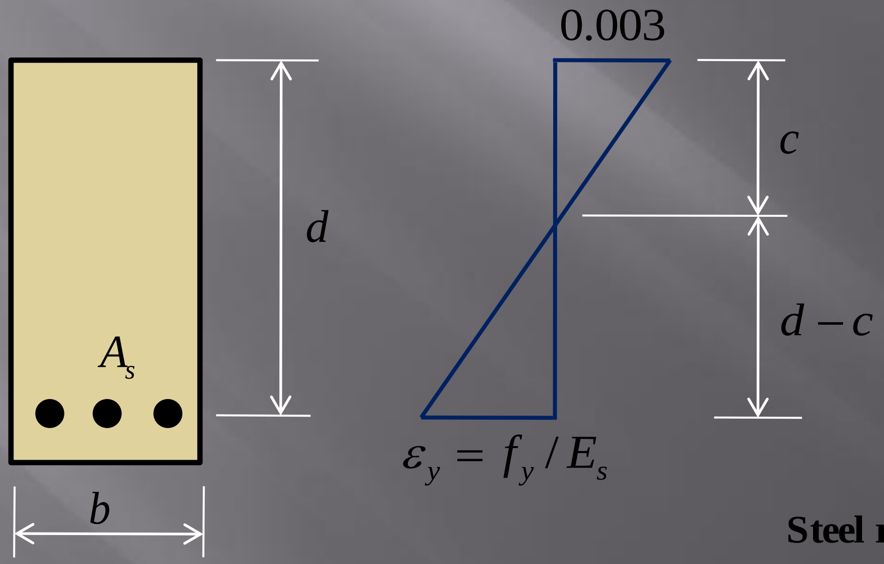
$\Rightarrow$ **Called Ductile or Tension controlled members**

$\rho > \rho_b \Rightarrow$ **Brittle failure i.e. concrete crushes before the steel yields**

$\Rightarrow$ **Called Brittle or Compression controlled members**

Balanced Section

- A section that has a steel ratio such that the steel reaches yield strain (f_y/E_s) when the concrete attains strain equal to 0.003.

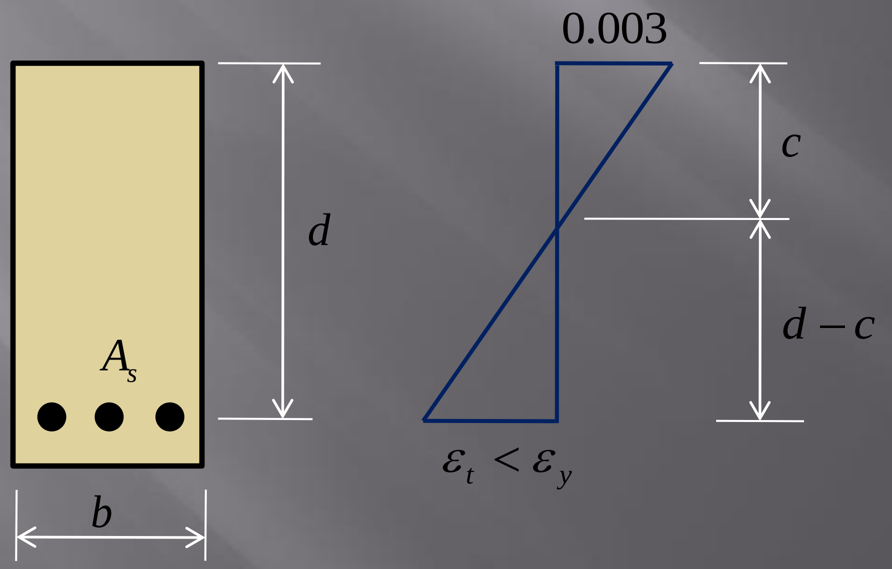


Steel ratio $\rho = \dfrac{A_s}{bd}$

For balanced section : $\rho = \rho_b$

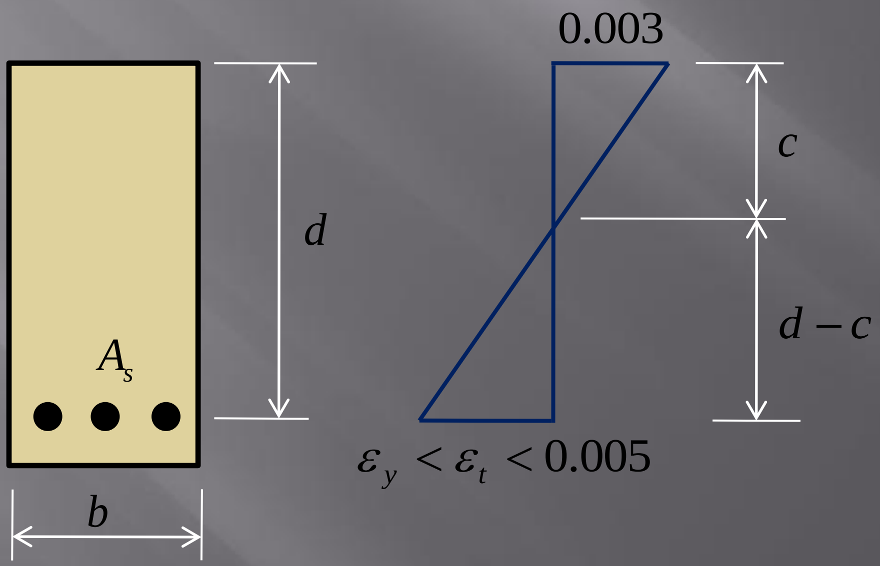
Brittle Members

- Members whose steel tensile strain is less than ε_y when the concrete strain reaches 0.003 are called compression-controlled – considered to be brittle. (ACI Section 10.3.4)
- Concrete crushes before steel yields.
- Deflections are small and there is little warning of failure.



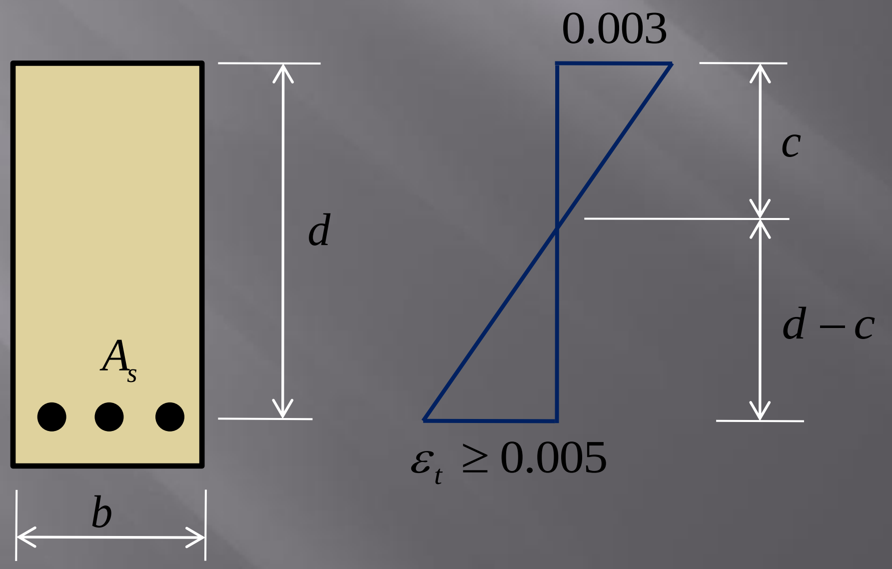
Transition Region

- Members with steel strains between ε_y and 0.005 are in a transition region
- For 420 MPa steel, ε_y can be approximated as 0.0021



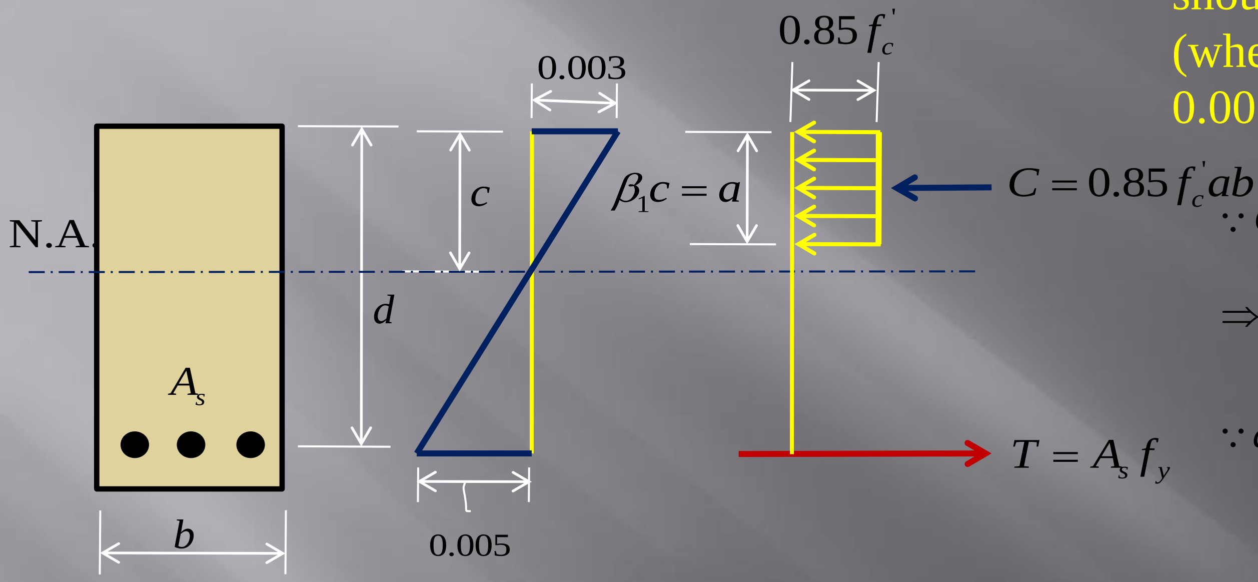
Ductile members

- Members whose steel tensile strain is greater than 0.005 when the concrete strain reaches 0.003 are called tension-controlled – considered to be fully ductile (ACI Section 10.3.4)
- Steel yields before concrete crushes
- Deflections are large and there is warning of failure



Maximum Steel Ratio

In order to have the member ductile enough steel tensile strain should not be less than 0.005 (when the concrete strain reaches 0.003).



$$\because C = T \Rightarrow 0.85 f_c^{'} ab = A_s f_y$$

$$\Rightarrow a = \frac{A_s f_y}{0.85 f_c^{'} b} = \frac{\rho f_y d}{0.85 f_c^{'}} \text{ where } \rho = \frac{A_s}{bd};$$

$$\because a = c\beta_1 \Rightarrow c = \frac{a}{\beta_1} = \frac{\rho f_y d}{0.85 \beta_1 f_c^{'}}$$

$$\frac{c}{d} = \frac{0.003}{(0.003 + 0.005)}$$

$$\frac{c}{d} = \frac{3}{8} \Rightarrow c = \frac{3}{8} d$$

$$\therefore \frac{\rho f_y d}{0.85 \beta_1 f_c^{'}} = \frac{3d}{8} \Rightarrow \rho = \left(\frac{0.85 \beta_1 f_c^{'}}{f_y} \right) \left(\frac{3}{8} \right)$$

$$\Rightarrow \boxed{\rho_{\max} = \left(\frac{0.85 \beta_1 f_c^{'}}{f_y} \right) \left(\frac{3}{8} \right)}$$

This is the maximum steel in order to have section fully ductile.

Minimum Percentage of Steel

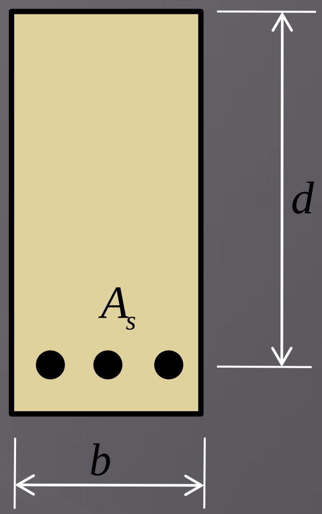
- Sometimes the applied bending moment (M_u) is so small that theoretically even a plane concrete section is able to resist it. However, If the ultimate resisting moment of the section is less than its cracking moment, the section will fail immediately when a crack occurs.
- This type of failure may occur without warning. To prevent such a possibility codes specify a certain amount of reinforcing that must be used at every section of flexural members.
- According to ACI (10.5.1):

$$A_{s,\min} = \left(\frac{\sqrt{f_c'}}{4 f_y}\right) b_w d \geq \left(\frac{1.4}{f_y}\right) b_w d$$

where b_w = web width of beams.

$$\because \rho_{\min} = \frac{A_{s,\min}}{b_w d} \Rightarrow \rho_{\min} = \left(\frac{\sqrt{f_c'}}{4 f_y}\right) \geq \left(\frac{1.4}{f_y}\right)$$

Steps in determining the design moment capacity



1a. Find steel ratio $\rho = \dfrac{A_s}{bd}$

1b. Find $\rho_{\min} = \left(\dfrac{\sqrt{f_c'}}{4 f_y}\right) \geq \left(\dfrac{1.4}{f_y}\right)$

1c. Find $\rho_{\max} = \left(\dfrac{0.85\beta_1 f_c'}{f_y}\right)\left(\dfrac{3}{8}\right)$

1d. If $\rho_{\min} < \rho < \rho_{\max}$ OK. Go to the next step.

2. Find $a = \dfrac{A_s f_y}{0.85 f_c' b}$ and $c = \dfrac{a}{\beta_1}$

3a. Find strain in tensile steel $\varepsilon_t = \dfrac{d-c}{c}(0.003)$

3b. If $\varepsilon_t > 0.005$ Section is tension controlled $\Rightarrow \phi = 0.90$

If $0.002 < \varepsilon_t < 0.005$ Section is in transition zone

$\Rightarrow \phi = 0.65 + (\varepsilon_t - 0.002)\left(\dfrac{250}{3}\right)$

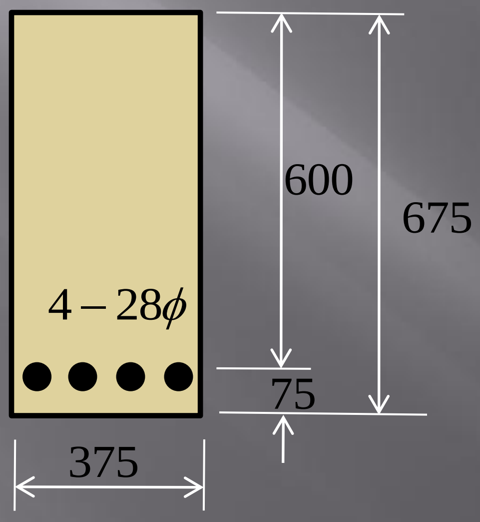
4. If section is tension controled or in transition zone

Design moment capacity $\phi M_n = \phi A_s f_y \left(d - \dfrac{a}{2}\right)$

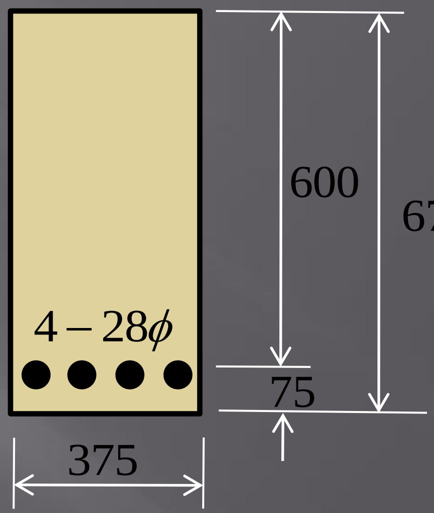
Note: If $\varepsilon_t < 0.004$ Section is not ductile enough. Section is not suitable.

Example

Determine the design moment capacity ϕM_n of the beam section shown in Figure below if $f_c' = 30$ MPa and $f_y = 420$ MPa.



$$\textbf{Steel ratio } \rho = \frac{A_s}{bd} = \frac{4\times\left(\frac{\pi}{4}\times 28^2\right)}{375\times 600} = \frac{2461.76}{225000} = 0.0109$$



$$\rho_{\min} = \left(\frac{\sqrt{f_c'}}{4f_y}\right) \geq \left(\frac{1.4}{f_y}\right) \Rightarrow \rho_{\min} = \left(\frac{\sqrt{30}}{4\times 420}\right) \geq \left(\frac{1.4}{420}\right)$$

$$\Rightarrow \rho_{\min} = \left(\frac{\sqrt{30}}{4\times 420}\right) \geq \left(\frac{1.4}{420}\right) = 0.0033 \geq 0.0033$$

$$\Rightarrow \rho_{\min} = 0.0033$$

$$\rho_{\max} = \left(\frac{0.85\beta_1 f_c'}{f_y}\right)\left(\frac{3}{8}\right) \Rightarrow \rho_{\max} = \left(\frac{0.85\times 0.85\times 30}{420}\right)\left(\frac{3}{8}\right)$$

$$\Rightarrow \rho_{\max} = 0.0193$$

$\because \rho_{\min} < \rho < \rho_{\max}$ OK

Solution Contd….

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2461.76 \times 420}{0.85 \times 30 \times 375} = 108.12 \text{ mm}$$

$$\because a = \beta_1 c \Rightarrow c = \frac{a}{\beta_1} = \frac{108.12}{0.85} = 127.20 \text{ mm}$$

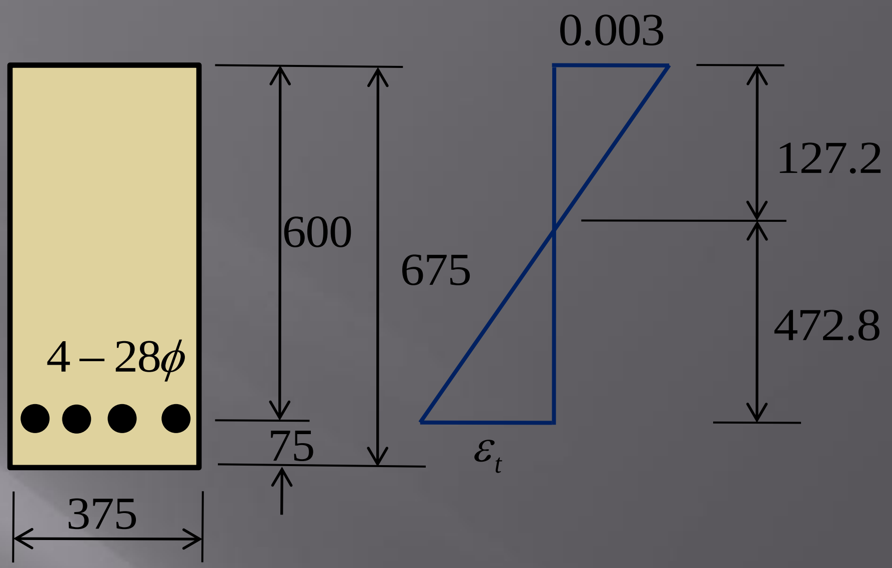
$$\varepsilon_t = \frac{d - c}{c}(0.003)$$

$$\Rightarrow \varepsilon_t = \frac{600 - 127.2}{127.2}(0.003) = 0.0111$$

$$\because \varepsilon_t = 0.0111 > 0.005$$

Section is tension controlled.

$$\Rightarrow \phi = 0.90$$



$$\therefore M_n = T\left(d - \frac{a}{2}\right) = A_s f_y \left(d - \frac{a}{2}\right)$$

$$\Rightarrow M_n = 2461.76 \times 420\left(600 - \frac{108.12}{2}\right)$$

$$\Rightarrow M_n = 564.46 \times 10^6 \text{ Nmm} \Rightarrow M_n = 564.46 \text{ kNm}$$

$\therefore$ Design moment capacity $\phi M_n = 0.9 \times 564.46$ kNm

$$\Rightarrow \phi M_n = 508.0 \text{ kNm}$$